量纲分析与单位

量纲分析

- 任何物理量的量纲都可以表示为基本量量纲的幂次乘积：$[Q] = L^a M^b T^c$
- 物理等式两边的量纲必须一致
- 量纲分析可以用来：
    i. 检验公式的正确性
    ii. 推导物理量之间的关系
    iii. 确定未知常数的量纲

例题

例1： 使用量纲分析法证明开普勒第三定律。

质点运动学

坐标系

笛卡尔坐标系

(x, y, z),基底 $\hat{i}, \hat{j}, \hat{k}$
位置向量：$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
速度：$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
加速度：$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

极坐标系

(r, θ)，基底 $\hat{r}, \hat{\theta}$

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}, \quad \frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

推导：

球坐标系

(r, θ, ϕ)，基底 $\hat{r}, \hat{\theta}, \hat{\phi}$

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

伽利略变换

$$\vec{r}' = \vec{r} - \vec{v}t$$

$$t' = t$$

例题

例1：初始时刻，狐狸处于原点，兔子在(R,0)处。此后，兔子沿圆心为原点，半径为R的圆，以恒定角速度ω运动。狐狸的速度大小始终为ωR，速度方向为两者连线。求狐狸的运动方程。

牛顿定律与非惯性系

惯性系与非惯性系

- **惯性系**：牛顿定律成立的参考系
- **非惯性系**：牛顿定律不成立的参考系，需要引入惯性力
- **惯性力**：$\vec{F}_{\text{惯}} = -m\vec{a}_{\text{系}}$，其中$\vec{a}_{\text{系}}$是非惯性系相对惯性系的加速度

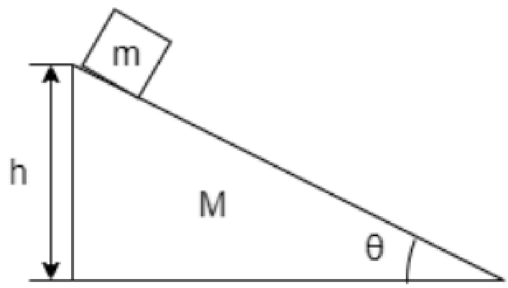
利用非惯性系解题

- 确定非惯性系的加速度
- 在研究对象上加上惯性力
- 按照惯性系的方法列方程求解

例题

例1：

A right triangular wedge of mass M, height h, angle θ, supports a cubic block of mass m on its side. Initially the wedge rest on a horizontal table, and the block is put at the highest point on the wedge side. Then the block is released from rest. Neglect the size of the block, the friction between the block and the wedge, and the friction between the wedge and the table.



question: Now we do NOT want the block to slide down the wedge as in 1 and 2. To achieve this, we can apply a suitable addtional horizontal force F to the wedge at the moment of releasing the block, such that the block will keep stationary relative to the wedge. Find the magnitude and the direction of this force.

例2：证明科里奥利力的大小为$F_{cor}=-2m\omega\times v$，其中$\omega$是旋转系的角速度，v是物体在旋转系中的速度。

动量

质心运动

- 质心位置：$\vec{r}_{cm}=\frac{\sum m_i\vec{r}_i}{M}$
- 质心速度：$\vec{v}_{cm}=\frac{\sum m_i\vec{v}_i}{M}$
- 质心运动定理：$M\vec{a}_{cm}=\sum\vec{F}_{外}$

变质量物体运动

小量形式的动量定律：

$$m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}=\vec{F}_{外}+\frac{\mathrm{d}m}{\mathrm{d}t}\vec{u}_{\mathrm{rel}}$$

其中$\vec{u}_{\mathrm{rel}}$是相对于物体的喷出物速度。

例题

例1：一艘宇宙飞船在太空中飞行，初始质量为M_0，以速度v_0飞行。飞船以恒定速率 m 喷出燃料，喷出物相对于飞船的速度为u，太空中有尘埃，密度为ρ，飞船的截面积为S。求飞船在时间t后的速度。

例2：

A string of length L was initially hung above the ground, where the top end was held by hand, with the bottom end almost touching the ground (as shown in Fig.1).At t=0 the top end was released from rest,and the string begins to fall freely. Asume the mass of the string is λ per unit length. We also assume the tension in the string is zero in other words, the part of the string that has been lying on the ground does not affect the remaining part of the string tha is falling freely.
(a) What is the speed v of the top end when it has fallen a distance h?
(b) Two forces are relevant for the string,namely, the gravitational force and the supportive force from the ground.For each force, state whether it is a conservative force,and explain why.
(c) Give an expression for the supportive force acting on the string by the ground when the string has fallen a distance h.

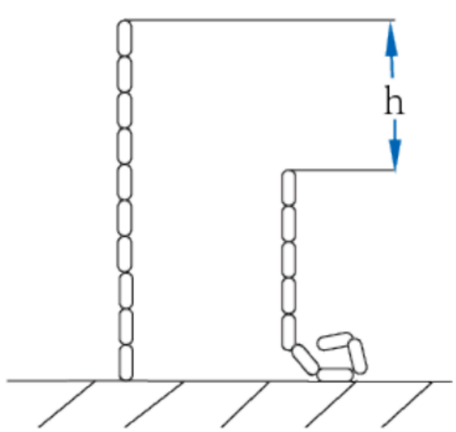


Figure 1: falling string

刚体力学

转动惯量

- 定义：$I = \sum m_i r_i^2 = \int r^2 dm$
- 常见物体的转动惯量：
  - 细杆（绕一端）：$I = \frac{1}{3}ML^2$
  - 细杆（绕中心）：$I = \frac{1}{12}ML^2$
  - 圆盘（绕中心）：$I = \frac{1}{2}MR^2$
  - 圆环（绕中心）：$I = MR^2$
  - 球体（绕中心）：$I = \frac{2}{5}MR^2$
  - 球壳（绕中心）：$I = \frac{2}{3}MR^2$
- 平行轴定理：$I = I_{cm} + Md^2$

力矩与转动定律

- 力矩：$\vec{\tau} = \vec{r} \times \vec{F}$
- 转动定律：$\sum \tau = I\alpha$

角动量

- 角动量：$\vec{L} = \vec{r} \times \vec{p}$（质点），$\vec{L} = I\vec{\omega}$（刚体绕定轴转动）
- 角动量定理：$\frac{d\vec{L}}{dt} = \sum \vec{\tau}_{外}$
- 角动量守恒：合外力矩为零时，$\vec{L} = \text{constant}$

转动动能与功

- 转动动能：$K = \frac{1}{2}I\omega^2$
- 力矩做功：$W = \int_{\theta_i}^{\theta_f} \tau d\theta$
- 刚体的机械能(柯尼希定理)：$E = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}^2 + U$

解题步骤

1. 选择旋转轴：优先质心
2. 首先作为质点，列平动方程
3. 列转动方程
4. 观察约束条件，列出转动与平动间的牵连方程
5. 联立方程求解

例题

例1：考虑地面摩擦，在一个物块从v减速到0的过程中，以地面一点为参考点，那么物块角动量是否守恒？为什么？

例2：

2 Rod hinged to a massive pulley (25 points)

Consider the configuration displayed in Figure 2.At the particular moment shown,the uniform rod of length L=0.8m and mass M_R=6.5kg is oriented horizontally and is rotating with an angular velocity ω=3.5 rad s^{-1}.One end of the rod is fixed, and the other end of the rod(point B)is attached to a massless string.The string wraps around a massive pulley of mass M_P=10.2kg and radius R_P=0.2m without slipping, and attached to other end of the string is a block of mass M_A=12.1kg. Treat the pulley as a uniform disc that rotates about the axis through P without friction,and take the acceleration due to gravity to be g=9.8m/s^2.
(a)Show that the moment of inertia I_P of the pulley about an axis through its center(point P),perpendicular to the circular cross section,is $I_P = \frac{1}{2}M_P {R_P}^2$.
(b)Show that the moment of inertia I_R of the rod about an axis at one end of the rod is $I_R = \frac{1}{3}M_R L^2$.
(c)Find the tangential component of the accerleration $a_T$of the end of the rod at point B and its direction.
(d)Find the radial component of the acceleration $a_R$of the end of the rod at point B and its direction.

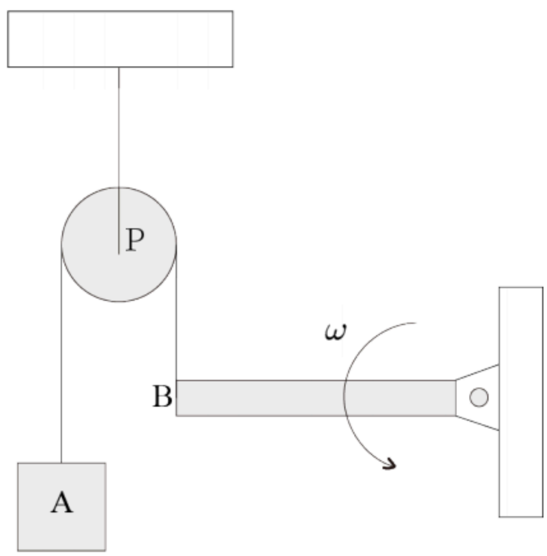


Figure 2: The hinged pulley system

例3：离心势能

一个质量为 m 的质点在有心力场中运动，受到大小为 kr,方向指向圆心的有心力。质点具有恒定的角动量大小 L 。

(1) 若质点做半径为 r_0 的匀速圆周运动，求 r_0 与 k 、 L 、 m 的关系；

(2) 若给质点一个小的径向扰动，质点将在径向做小振动。试用有效势能（即离心势能与势能之和）的方法，求该小振动的周期 T 。

FROM: ZJU-GPA

Angular speed $\boldsymbol{\omega} = \mathrm{d}\boldsymbol{\theta}/\,\mathrm{d}t$

Angular acceleration $\boldsymbol{\alpha} = \mathrm{d}\boldsymbol{\omega}/\,\mathrm{d}t$

Resultant torque $\sum \boldsymbol{\tau} = I\boldsymbol{\alpha}$

If

$$\boldsymbol{\alpha} = \text{constant} \begin{cases} \boldsymbol{\omega}_f = \boldsymbol{\omega}_i + \boldsymbol{\alpha} t \\ \boldsymbol{\theta}_f - \boldsymbol{\theta}_i = \boldsymbol{\omega}_i t + \frac{1}{2}\boldsymbol{\alpha} t^2 \\ {\omega_f}^2 = {\omega_i}^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$$

Work $W = \int_{\theta_i}^{\theta_f} \tau \,\mathrm{d}\theta$

Rotational kinetic energy $K_{\mathrm{R}} = \frac{1}{2} I\omega^2$

Power $P = \boldsymbol{\tau} \cdot \boldsymbol{\omega}$

Angular momentum $\boldsymbol{L} = I\boldsymbol{\omega}$

Resultant torque $\sum \boldsymbol{\tau} = \mathrm{d}\boldsymbol{L}/\,\mathrm{d}t$

Linear speed $\boldsymbol{v} = \mathrm{d}\boldsymbol{x}/\,\mathrm{d}t$

Linear acceleration $\boldsymbol{a} = \mathrm{d}\boldsymbol{v}/\,\mathrm{d}t$

Resultant force $\sum \boldsymbol{F} = m\boldsymbol{a}$

If

$$\boldsymbol{a} = \text{constant} \begin{cases} \boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a} t \\ \boldsymbol{x}_f - \boldsymbol{x}_i = \boldsymbol{v}_i t + \frac{1}{2}\boldsymbol{a} t^2 \\ {v_f}^2 = {v_i}^2 + 2a(x_f - x_i) \end{cases}$$

Work $W = \int_{x_i}^{x_f} F_x \,\mathrm{d}x$

Kinetic energy $K = \frac{1}{2} m v^2$

Power $P = \boldsymbol{F} \cdot \boldsymbol{v}$

Linear momentum $\boldsymbol{p} = m\boldsymbol{v}$

Resultant force $\sum \boldsymbol{F} = \mathrm{d}\boldsymbol{p}/\,\mathrm{d}t$